Problems: Celestial mechanics I: solar system and exoplanets

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You may use the following constants:

Earth's distance from the Sun – $R_\oplus = 1\,\mathrm{AU} = 1.496 \times 10^{11}\,\mathrm{m}$,
Earth's orbital speed – $v_\oplus = 30\,\mathrm{km/s}$,
Earth's radius – $r_\oplus = 6400\,\mathrm{km}$,
Gravitational acceleration on Earth – $g = 9.81\,\mathrm{m/s^2}$.

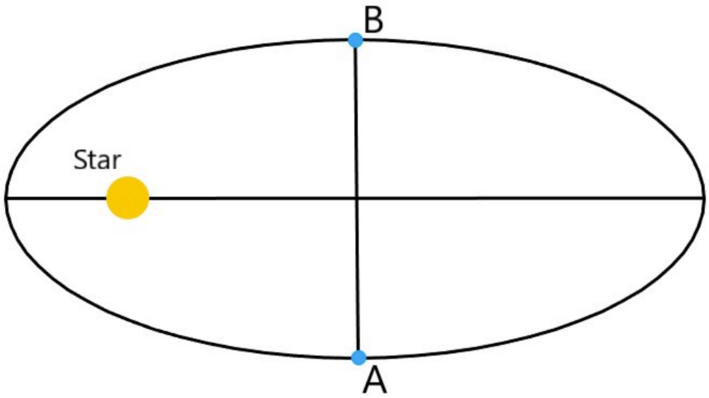
Celestial mechanics basics

1. Solve the exercises on the cheat sheet.

2. A Sun-orbiting periodic comet is the farthest at $r_1 = 31.5\,\mathrm{AU}$ and the closest at $r_2 = 0.5\,\mathrm{AU}$. What is the orbital period of this comet? What is the area per year in AU^2/year that the comet sweeps?

3. A body orbiting the Sun at $R = R_\oplus$ is given a radial velocity v_r. How big must it be for the body to exit the gravitational influence of the Sun?

4. Consider the following elliptical orbit of a comet around a star (orbiting counter-clockwise). If the total period of the orbit is T and eccentricity e, how much time does it take for it to reach from A to B?



5. A meteorite that is radially approaching the Earth collides with a space station of equal mass that revolves around the Earth in a circular orbit with radius R. For all parts of the question, express your results in terms of the mass M of the Earth, the gravitational constant G and the mass of the meteorite is m.

Assume that, after the impact, the meteorite and the space station form a conglomerate that moves in a closed orbit which approaches the center of the Earth at a minimum distance $R/2$. What is the:

(i) The shape of the trajectory
(ii) the speed of the meteorite just before the collision,
(iii) the minimum and maximum speeds of the conglomerate,

6*. An asteroid approaches the Sun from far away with an impact parameter b and speed v_0. How close does the asteroid get to the Sun, and by how big of an angle does it get deflected after leaving the gravitational influence of the Sun?

7*. We aim to make a space probe launched from Earth leave the Solar System with the help of a single gravitational slingshot, which utilises the relative movement and gravity of one of the planets that orbit the Sun. In astronomical units, how far from the Sun would the "ideal" planet be if the initial launch speed of the probe is to be kept to a minimum?

When solving the problem, make the following approximations:

- The orbits of the planets are circles all lying in the same plane.
- Near a planet, it is sufficient to take into account only the gravity due to that planet.
- Far from all planets, only the Sun's gravity is relevant.

Does a real planet exist at or near the optimal orbit?

Lagrange points

8. As a first order approximation, how far is L_2 from Earth?

Exoplanets

9. TRAPPIST-1d is a temperate exoplanet that orbits the ultra-cool M dwarf star TRAPPIST-1 with a semi-major axis of 0.022 AU. TRAPPIST-1 has a mass of 0.089 Solar masses and an effective temperature of 2511 K. Through transit timing variations induced by other planets in the TRAPPIST-1 system, TRAPPIST-1d is estimated to have a mass of 0.420 Earth masses. Assuming that TRAPPIST-1d has a circular orbit (which is a good approximation because the measured eccentricity is only 0.008), what is the radial velocity semi-amplitude of TRAPPIST-1 due to the orbital motion of TRAPPIST-1d, in m/s?